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Anomaly inflow on orientifold planes

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Abstract: We examine some six-dimensional orientifold models with $N = 1$ supersymmetry, which can be realised as intersecting 7-branes and 7-planes. These models are studied in the light of recent work showing that orientifold planes carry anomalous gravitational couplings on their world-volume. We show that gravitational anomalies can be locally cancelled by these new couplings at every point in the internal space, under the assumption that the anomaly residing on orientifold planes is distributed in a particular way among brane-plane and plane-plane intersections.

Keywords: String Duality, Branes in String Theory, M- and F-theories and Other Generalizations, Anomalies in Field and String Theories.

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Contents

1 Introduction

Two types of extended objects have played an important role in string theory in recent years: Dirichlet branes and orientifold planes. Classically, there are well-known differences between them: in contrast to D-branes, orientifold planes are non-dynamical and do not carry Yang-Mills multiplets on their world volume. However, both kinds of objects are charged under appropriate p-form potentials. Moreover, in certain models, when quantum corrections are taken into account, Z_2 orientifold planes can split into nonperturbative generalizations of D-branes [1, 2], so dynamically there is something in common between these two types of objects.

It has been shown recently that Z_2 orientifold planes behave very much like Dirichlet branes as far as WZ gravitational couplings are concerned [3]. Indeed, both branes and planes carry certain precise gravitational Wess-Zumino terms on their world-volumes. For the case of branes, these terms are derived by taking a pair of intersecting branes and requiring cancellation of gravitational anomalies on the intersection region via inflow from the branes [4]. The analogous terms on orientifold planes were derived in a different way [3], so turning the logic around, one should be able to check that the predicted WZ terms on planes actually cancel the anomalies on their intersection regions with D-branes and with other planes.

As we will see, it will not be possible to actually demonstrate that this local anomaly cancellation does take place. Instead, we will find a prediction for how the WZ terms on intersecting branes and planes should be distributed among the brane-plane and plane-plane intersections in order to bring about local anomaly cancellation. An independent verification of our prediction would be useful in demonstrating that anomalies really are cancelled locally by the gravitational couplings found in ref. [3].

2 Anomalies on intersections of branes and planes

Let us assume that the gravitational WZ couplings on branes and planes are of the form

$$\int_B C \wedge Y(R)\,, \tag{2.1}$$

where C is the RR background, $Y(R)$ is some curvature polynomial and B is the world-volume dimension. Generalizing ref. [4], we take a configuration of two 7-branes/planes intersecting over a six dimensional space B_{12}. Thus we are considering brane-brane, brane-plane and plane-plane intersections. The WZ coupling now looks like

$$-\sum_{i=1}^{2}\int_i \left(G_1 \wedge Y_7 + G_5 \wedge Y_3 + Y_0 \wedge {}^*C_0\right). \tag{2.2}$$

Here $i = 1, 2$ labels the world-volume of the two intersecting objects, each of which can be a 7-brane or a 7-plane. C_n and Y_n are the background RR n-form and curvature n-form respectively.

In the presence of branes or planes, G_n and dC_{n-1} differ. The former is gauge invariant whereas the latter is not. Hence a partial integration has been carried out to get eq. (2.2) from eq.(2.1), using $Y_8 \equiv dY_7$, $Y_4 \equiv dY_3$.

The relevant Bianchi identities and equations of motion are:

$$\begin{aligned} dG_1 &= \delta^2(1)Y_0(1) + \delta^2(2)Y_0(2) \\ dG_5 &= \delta^2(1)Y_4(1) + \delta^2(2)Y_4(2) \\ d*G_1 &= \delta^2(1)Y_8(1) + \delta^2(2)Y_8(2)\,. \end{aligned} \tag{2.3}$$

From the last equation we see that ${}^*C_0$ has an anomalous variation

$$\delta(*C_0) = -\delta^2(1)Y_6(1) - \delta^2(2)Y_6(2)\,, \tag{2.4}$$

where δ is a general coordinate transformation, and $\delta Y_7 = dY_6$. Therefore, under a general coordinate transformation, the WZ terms undergo an anomalous variation which can be shown to follow by the descent procedure from

$$-2\int \left[Y_8(1)Y_0(2) + Y_8(2)Y_0(1) + Y_4(1)Y_4(2)\right]. \tag{2.5}$$

Thus the anomalies on the intersection regions will be proportional to the product of the corresponding curvature forms for brane-brane (BB), brane-plane (BP) and plane-plane (PP) intersections.

3 Anomaly cancellation

The WZ terms on a 7-brane have been determined earlier in [4, 5, 6]. The result is as follows:

$$(WZ)_B = \int_{\Sigma^8} \left[{}^*\widetilde{\phi} - \frac{1}{48}C^{(4)+} \wedge p_1 + \frac{1}{23040}\widetilde{\phi} \wedge (9p_1^2 - 8p_2)\right], \tag{3.1}$$

where $\widetilde{\phi}$ and $C^{(4)+}$ are the RR 0-form and 4-form potentials of the type IIB string, and p_i are the Pontryagin classes given in terms of the curvature form R. The terms on the orientifold plane have been worked out in [3]. In this case the result is

$$(WZ)_P = \int_{\Sigma^8} \left[-4^* \widetilde{\phi} - \frac{1}{24} C^{(4)+} \wedge p_1 + \frac{1}{11520} \widetilde{\phi} \wedge (27p_1^2 - 44p_2) \right]. \tag{3.2}$$

The first term in the above equations determine the charges of the branes and planes. As for the second term in $(WZ)_P$, a different argument for its coefficient can be given as follows(this is in the spirit of ref. [7]). Let us take type IIB on a T^2/Z_2 orientifold. Consistency conditions require the existence of 4 orientifold planes and 16 D-branes. This theory is equivalent to F-theory on K3 [8, 1].

Now, F-theory is conjectured to have a 12-dimensional term of the form $\int C^{(4)+} \wedge I_8$ [9, 10], where I_8 is an 8-form polynomial in the curvature. Compactifying this on K3 gives a term proportional to $\int C^{(4)+} \wedge p_1$ in 8 dimensions. On the IIB side, let the branes and planes carry the terms $\alpha \int C^{(4)+} \wedge p_1$ and $\beta \int C^{(4)+} \wedge p_1$ respectively. Since we know that $\alpha = -\frac{1}{48}$, comparing the IIB result to that from F-theory gives $\beta = -\frac{1}{24}$. In other dimensions, $C^{(4)+}$ is replaced by different RR fields, and one can show that the WZ couplings (in various dimensions $d = 10 - n$) of branes and planes come with factors α and β, where α and β are related by $32\alpha + 2^{n+1}\beta = -1$. The specific case of $d = 7$ was treated explicitly in ref. [7].

The third terms in the above equations for the brane and plane are required to satisfy the conjectured duality of IIB on T^2/Z_2 to the heterotic string on T^2 [8, 1]. This duality implies the existence of a term $\phi \wedge X_8$ on the heterotic side, where X_8 is another 8-form polynomial in the curvature. (X_8 actually depends on the gauge field strengths as well, but since planes have no gauge couplings, the gauge fields are set equal to zero for this discussion.) Since IIB has no such term (its existence would violate SL(2,Z) invariance), it must come from branes and planes, as has been shown in ref. [3].

Before we go on to calculate the inflow contributions I_{BB}, I_{BP} and I_{PP}, we should ask what anomalies they are expected to cancel. What is already known [4] is that the inflow I_{BB} onto brane-brane intersections cancels the anomalies of the hypermultiplets which come from Dirichlet-Neumann (DN) open strings connecting two intersecting branes. In order to investigate anomalies on BP and PP intersections we need to embed these in a definite orientifold model, unlike BB intersections, which can be analyzed independent of a specific model. We examine the Gimon-Polchinski (GP) orientifold [11][1] and the Blum-Zaffaroni-Dabholkar-Park (BZDP) orientifold model [13, 14] to illustrate the cancellation process explicitly.

3.1 GP orientifold

Since we want a model with intersecting 7-branes and 7-planes, we consider a T-dual version [15] of the GP orientifold. This is defined as a $Z_2 \times Z_2'$ orientifold of IIB on T^4,

[1] This model and related ones were studied earlier as open string orbifolds, in ref. [12].

where $Z_2 = \{1, g\}$, $g = \mathcal{I}_{67}(-1)^{F_L}\Omega$ and $Z'_2 = \{1, h\}$, $h = \mathcal{I}_{89}(-1)^{F_L}\Omega$. T^4 is a four dimensional torus labelled by $(x^6, ..., x^9)$, Ω is orientation reversal, and $\mathcal{I}_{ij}$ is reversal of the space dimensions x^i, x^j.

We now have two sets of orientifold planes, one set at the fixed points of $\mathcal{I}_{67}$ and the other set at the fixed points of $\mathcal{I}_{89}$. There are four orientifold planes in a set, with each plane carrying a charge of -4 units of the RR scalar $\widetilde{\phi}$. Cancellation of charges requires placing 16 D 7-branes transverse to one plane and another set of 16 D 7-branes transverse to an orthogonal plane. The charge is neutralized locally when each orientifold plane has four D-branes on top of it. Additionally, $gh = \mathcal{I}_{6789}(-1)^F$ will give rise to orbifold twisted sector states.

We will show that in this model, anomaly inflow onto PP and BP intersections is necessary to locally cancel the anomalies which come from the untwisted sector, the orbifold twisted sector and the brane world-volumes. It will turn out that a specific distribution of the anomaly on these two types of intersections brings about local anomaly cancellation; this can perhaps be tested independently in the future.

Anomaly cancellation in this model can be viewed in two ways: globally and locally. Globally, due to overall charge cancellation, there will be no anomaly inflow and the bulk anomalies will cancel among themselves. Thus the branes and planes contribute anomaly inflow to the intersection region in such a way that

$$n_{BB}I_{BB} + n_{BP}I_{BP} + n_{PP}I_{PP} = 0\,, \tag{3.3}$$

where n_{BB}, n_{BP}, n_{PP} are the number of brane-brane, brane-plane and plane-plane intersections.

The other aspect, local cancellation, is the emphasis of this paper. WZ terms on branes are believed to ensure local anomaly cancellation [4]. In the spirit of the idea that planes (though not dynamical classically) behave very much like branes, we expect the analogous result to go through for BP and PP intersections.

A similar situation occurs in the orientifold of M-theory on T^5/Z_2 [16, 17]. On one hand, due to cancellation of charge, there is actually no inflow – under a general coordinate transformation the Lagrangian remains invariant. On the other hand, as observed in ref. [17], the anomalies in the theory are cancelled locally by inflow from the bulk due to the $C \wedge I_8$ term in the action. The presence of five-branes activates the inflow *and* contributes 16 tensor multiplets to the spectrum. This inflow is reversed by planes carrying *minus* half a unit of charge.

One important point about anomaly cancellation in the GP model is that it is sufficient to cancel the irreducible part of the anomaly, as the reducible part can be cancelled by an extension of the Green-Schwarz mechanism [18]. Moreover, for our purposes we can ignore the observations in ref. [18] about non-perturbative effects breaking some $U(1)$ factors, since those issues are irrelevant for cancellation of the irreducible part of the anomaly. Hence we will list the perturbative spectrum in what follows.

The spectrum of the T-dual version of the GP model arises as follows. The untwisted sector consists of one gravity multiplet, one tensor multiplet and four hypermultiplets of $D = 6, N = 1$ supersymmetry. The twisted sector (coming from both open strings and orbifold twisted-sector states) consists of vector multiplets and hypermultiplets. The total spectrum in various regions of the moduli space is one supergravity multiplet $(g_{\mu\nu}, B^+_{\mu\nu}, \psi_\mu)$, one tensor multiplet $(B^-_{\mu\nu}, \chi^R, \phi)$, vector multiplets (A_μ, ψ^L) in the adjoint representation of the enhanced gauge group G at various points in the moduli space, plus hypermultiplets $(4\phi, \psi^R)$ in various representations. We list the hypermultiplets along with their origin in the following table.

Group $G \times G'$	$U(16) \times U(16)'$	$U(4)^4 \times U(4)'^4$	$U(2)^8 \times U(2)'^8$
16 fixed points of gh	$16 \times (\mathbf{1}, \mathbf{1})$	$16 \times (\mathbf{1}, \mathbf{1})$	$16 \times (\mathbf{1}, \mathbf{1})$
antisym. rep. of G	$2 \times (\mathbf{120}, \mathbf{1})$	$8 \times (\mathbf{6}, \mathbf{1})$	$16 \times (\mathbf{1}, \mathbf{1})$
antisym. rep. of G'	$2 \times (\mathbf{1}, \mathbf{120})$	$8 \times (\mathbf{1}, \mathbf{6})$	$16 \times (\mathbf{1}, \mathbf{1})$
DN open strings	$1 \times (\mathbf{16}, \mathbf{16})$	$16 \times (\mathbf{4}, \mathbf{4})$	$64 \times (\mathbf{2}, \mathbf{2})$
untwisted sector	$4 \times (\mathbf{1}, \mathbf{1})$	$4 \times (\mathbf{1}, \mathbf{1})$	$4 \times (\mathbf{1}, \mathbf{1})$

Table 1: Hypermultiplets

The DN open string modes are treated separately, as the anomaly from them is cancelled by inflow from the branes to the intersection region [4]. Summing over the remaining multiplets at any point in the moduli space, we find that the irreducible part of the anomaly (the coefficient of $\mathrm{tr}R^4$) is equal to $\frac{2}{45}$.

Now we can calculate the inflow contribution from the branes and planes to the intersection region. Combining eqs.(2.5),(3.1) and (3.2), the result for the irreducible part of the anomaly inflow is:

$$I_{BB} = \frac{1}{5760}, \quad I_{BP} = \frac{7}{11520}, \quad I_{PP} = -\frac{44}{5760}\,. \tag{3.4}$$

Also, it is easy to see that in eq. (3.3), the relevant values are

$$n_{BB} = 256\,, \quad n_{BP} = 128\,, \quad n_{PP} = 16\,, \tag{3.5}$$

satisfying the consistency condition required by charge cancellation.

Using the above results one sees that

$$128\ I_{BP} + 16\ I_{PP} = -\frac{2}{45}\,, \tag{3.6}$$

which cancels the anomaly from the spectrum (excluding the DN open string modes) at all points in the moduli space. Note that the last term in eq.(3.2) actually does not contribute to this result, hence global anomaly cancellation does not rely on the existence of this term.

As we will see, for anomalies to be cancelled locally, the coefficient of the last term in eq.(3.2) gets correlated with the proportion in which bulk anomalies are distributed in

the orientifold model. Let us now examine how the anomalies from the various multiplets are distributed to the brane-brane, brane-plane and plane-plane intersection regions. We make the following observations:
(a) For the BB case, as noted above, the inflow cancels the anomaly from modes of DN open strings connecting the two branes. In other words the hypermultiplets in the (a,b) representation of the group $U(a)\times U(b)'$ lie on this intersection region. Overall at any point there are 256 hypers, contributing an anomaly of $-\frac{2}{45}$.
(b) There are 16 twisted sectors (from the gh part of the orientifold group) contributing 16 neutral hypermultiplets. These are constrained to lie one on each of the 16 plane-plane (PP) intersection regions. One way to confirm this is to go to the quantum corrected picture of this model. As has been shown in ref. [15], the PP intersection region joins smoothly to form a (nonperturbative) brane. This is due to the presence of blowup modes of the orbifold fixed points. These twisted sectors contribute a total anomaly of $16.-\frac{1}{5760}=-\frac{1}{360}$.
(c) The vectors and the hypers (which come from the multiplets on the branes) are confined along the BP intersection regions. The anomaly from these should be cancelled by inflow from the intersecting brane and plane. It is easy to check that the difference between the number of vectors and hypers is 32 at every point in the moduli space, hence the anomaly from these states is $32.\frac{1}{5760}=\frac{1}{180}$.
(d) At no point in the moduli space can a single brane move freely. The minimum number of branes that can move together in this theory is two, giving the gauge group $U(2)^8\times U(2)'^8$ [15]. In this case the anomaly from BB will be four times the single BB value, and the anomaly from BP will be double the original value.
(e) The only states not accounted for so far are the "bulk" multiplets, from the untwisted sectors, consisting of 1 gravity + 1 tensor + 4 hypers. They contribute a total anomaly of $\frac{1}{24}$. Because there is no anomaly in the 10d bulk theory, we must assume that this anomaly, which arises from the orientifolding operation, is distributed in some way over the orientifold planes.[2] This means that it can live on either the BP or the PP regions. Below we will discover in what proportion it must be distributed for local anomaly cancellation to take place.

From the above, the total anomalies at the BP and the PP intersection regions are $\frac{1}{180}$ and $-\frac{1}{360}$ respectively. From eqs. (3.4), (3.5) we know that the inflow contributions to the BP and PP regions are $\frac{7}{90}$ and $-\frac{11}{90}$ respectively. Therefore local anomaly cancellation demands that the untwisted sector multiplets, whose total anomaly is $\frac{1}{24}$, must have this anomaly distributed in the proportion $-\frac{1}{12}$ and $\frac{1}{8}$ to the BP and PP intersection regions. Since there are 128 BP and 16 PP intersections, this in particular implies that each individual BP intersection receives an anomaly of $-\frac{1}{1536}$ while each PP intersection receives $\frac{1}{128}$.

Thus, in the process of arguing that the WZ terms of ref. [3] are consistent with local anomaly cancellation, we have also made a prediction: the untwisted sector in the

[2] An analogous assumption was made in ref. [17], where the bulk anomaly was distributed equally over 32 orientifold 5-planes.

(T-dual) GP orientifold must have its anomaly localized on BP and PP intersections in the ratio $-2:3$. An independent confirmation of this prediction would provide significant support for the idea that anomalies are locally cancelled in these models.

3.2 BZDP orientifold

Next we consider a different model in six dimensions with $N=1$ supersymmetry, which can also be realised in terms of intersecting 7-branes [13, 14]. This model has the same orientifold group $Z_2 \times Z_2'$ as the GP model considered earlier, but the orientation-reversal symmetry Ω acts with an additional minus sign on the twisted sector states of the orbifold. (This is like turning on discrete torsion in the orbifold construction [19, 20]). This symmetry of the orbifold flips the sign of the twist fields at all fixed points.

The untwisted sector is the same as before, but now there are no charged hypermultiplets. They are all projected out. However, the orbifold twisted sectors contribute 16 tensor multiplets of $N=1$ supersymmetry. The vector multiplets are in the adjoint of $SO(8)^4 \times SO(8)'^4$. As this model has no hypers (from the branes) the moduli of moving the branes are also missing. We now have 2 sets of four intersecting orientifold planes at the fixed points of the orientifold group. A set of four D-branes lie on top of each orientifold plane. Therefore we have the following situation. There are 16 intersection regions. Each region consists of one PP, 16 BB and 8BP intersections. Now one has to calculate the total anomaly from each such point. The answer is

$$16 I_{BB} + 8 I_{BP} + I_{PP} = 0\,, \tag{3.7}$$

from eq. (3.4). Hence in this model there is no net inflow from the branes and planes.

The bulk anomalies cancel among themselves, as the spectrum satisfies the six dimensional anomaly cancellation equation:

$$H - V = 273 - 29T\,, \tag{3.8}$$

with $H=4, V=224$ and $T=17$.

Local cancellation in this model is easy to see from the following observations:

(1) The 16 intersection regions have a tensor multiplet each. These tensor multiplets take the place of hypers in the GP model, because here the hypers are projected out and tensors are retained.

(2) The vector multiplets which are in the adjoint of $SO(8)^4 \times SO(8)'^4$ should lie along the BP intersection region. But the model has no such *distinct* regions. Nevertheless one can assume the vectors to be distributed equally to the 16 intersection regions of the model, as each one contains a single BP intersection.

(3) For local cancellation of anomalies, an equal fraction of the total anomaly from the untwisted sector must go to each of the 16 symmetric intersection regions. Thus we need an anomaly of $\frac{1}{16}\cdot\frac{1}{24}$ at each intersection region. This also follows from our analysis of the GP model. Recall that there we predicted that the untwisted sector contributes an anomaly of $-\frac{1}{1536}$ to each BP intersection and $\frac{1}{128}$ to each PP intersection. Since each

intersection region in the BZDP model has 8 BP and 1 PP intersections overlapping, the anomaly from the untwisted sector will be $8. -\frac{1}{1536} + \frac{1}{128} = \frac{1}{384}$ on each such region, as expected.

4 Conclusions

The models we have studied here not only exhibit global cancellation of gravitational anomalies, but are also consistent with local anomaly cancellation on each of the defect regions.

We have argued that local anomaly cancellation will take place if anomalies are distributed in the specific ratio $-2:3$ on brane-plane and plane-plane intersections. Since we have not found an independent way of computing this distribution in the present models, our results do not actually prove that local anomaly cancellation does take place, but rather should be viewed as a new prediction for the way anomalies reside on brane-plane and plane-plane intersections.

Although we have only checked two models explicitly, we expect that all other 6-dimensional orientifold models[3] will exhibit local anomaly cancellation in the same way.

Clearly it is important to find an independent way of predicting the distribution of anomalies onto different types of defect intersections. This would confirm that the results of ref. [3] about anomalous couplings on orientifold planes are actually responsible for local anomaly cancellation. Perhaps more important, it would give some new insight into the orientifolding procedure itself, since we do not understand the detailed mechanism by which potential anomalies are created by orientifolding and eventually cancelled by inflow from the bulk.

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